

Cluster formation and the Sunyaev-Zel’dovich power spectrum in modified gravity: the case of a phenomenologically extended DGP model

Tsutomu Kobayashi¹ and Hiroyuki Tashiro²

¹*Department of Physics, Waseda University, Okubo 3-4-1, Shinjuku, Tokyo 169-8555, Japan*

²*Institut d’Astrophysique Spatiale (IAS), Bâtiment 121, F-91405, Orsay, France;
Université Paris-Sud 11 and CNRS (UMR 8617)*

26 May 2009

ABSTRACT

We investigate the effect of modified gravity on cluster abundance and the Sunyaev-Zel’dovich angular power spectrum. Our modified gravity is based on a phenomenological extension of the Dvali-Gabadadze-Porrati model which includes two free parameters characterizing deviation from Λ CDM cosmology. Assuming that Birkhoff’s theorem gives a reasonable approximation, we study the spherical collapse model of structure formation and show that while the growth function changes to some extent, modified gravity gives rise to no significant change in the linear density contrast at collapse time. The growth function is enhanced in the so called normal branch, while in the “self-accelerating” branch it is suppressed. The Sunyaev-Zel’dovich angular power spectrum is computed in the normal branch, which allows us to put observational constraints on the parameters of the modified gravity model using small scale CMB observation data.

Key words: cosmology: theory – large-scale structure of the universe

1 INTRODUCTION

General Relativity is surely the most successful theory of gravity that passes accurate tests in the solar system and laboratories. However, current cosmological observations indicate the presence of dark matter and dark energy; a large fraction of the Universe is made of unknown components. The mystery of the dark components is based on general relativity, and hence it tells us that what we do not know may be the long distance behaviour of gravity rather than the energy-momentum components in the Universe. In this sense, cosmological observations open up a new window to study the properties of gravity on large scales.

There are various alternative theories of gravity leading to interesting cosmological consequences. For example, the Dvali-Gabadadze-Porrati (DGP) model (Dvali et al. 2000) is one of the extra dimensional scenarios that can account for cosmic acceleration without introducing dark energy. The $f(R)$ theories, which modify the four-dimensional Einstein-Hilbert action explicitly, also realise the accelerated expansion of the Universe (Sotiriou & Faraoni 2008). MOND (Milgrom 1983) and its relativistic extension (Bekenstein 2004) explain galactic rotation curves without need for dark matter. A more phenomenological way of changing gravity is to assume Yukawa-like modification to a gravitational potential. Such modification yields effec-

tively a scale-dependent Newtonian constant, and its effect on the evolution of large scale structure has been investigated (Sealfon et al. 2005; Shirata et al. 2005, 2007; Stabenau & Jain 2006; Martino et al. 2008).

One of the powerful ways for distinguishing modified gravity from the Λ CDM model is to study the growth function because the modified growth function would leave its footprints on the cosmological large-scale structure. In this context, many authors have studied the integrated Sachs-Wolfe effect and weak lensing in modified gravity, and obtained observational constraints on modified gravity models. For instance, Schmidt (2008) investigated the effect of $f(R)$ gravity, the DGP model, and tensor-vector-scalar theory on weak lensing, and showed that for detecting signatures of modified gravity the weak lensing observation is a better probe than the integrated Sachs-Wolfe effect measured via the galaxy-CMB cross-correlation. Thomas et al. (2008) put constraints on the DGP model by using weak lensing data (CFHTLS-wide) combined with baryon acoustic oscillations and supernovae data. Schmidt et al. (2009) have studied the statistical properties of dark halos in $f(R)$ gravity by employing numerical simulations.

In this paper, we consider a generalisation of the DGP model, adding a term $\pm H^{2\alpha}/r_c^{2(1-\alpha)}$ in the Friedmann equation, where α and r_c are the model param-

ters (Dvali & Turner 2003; Koyama 2006; Afshordi et al. 2008; Khoury & Wyman 2009). The generalised model reduces to the self-accelerating and normal branches of the DGP model (with a cosmological constant) by taking $\alpha \rightarrow 1/2$, and reproduces the Λ CDM model in the $\alpha \rightarrow 0$ limit. Afshordi et al. (2008) studied cosmological perturbations in this modified gravity theory under several assumptions, and discussed various observational consequences of the model. Khoury & Wyman (2009) performed N-body simulations in the same model recently. We shall work on a different approach in the present paper: assuming that Birkhoff's theorem gives a reasonable approximation (as implied by Koyama (2006)), we study the spherical collapse model of structure formation with the modified Friedmann equation. Using the spherical collapse scenario, we can compute the Sunyaev-Zel'dovich (SZ) angular power spectrum through the Press-Schechter formalism (Press & Schechter 1974). Since the SZ angular power spectrum is sensitive to the distribution of dark halos, it is a good probe of modified distribution of dark halos in the generalised DGP model.

Before closing the introduction we should note that in any case modified gravity must satisfy solar system and laboratory tests, and some $f(R)$ models and the original DGP model indeed have mechanisms to reproduce ordinary gravity around the Sun and on the Earth. This point is not clear in the present model and is beyond the scope of the paper. We only remark that our Friedmann equation reduces to the ordinary one at high densities.

This paper is organized as follows. In the next section, we describe our modified gravity model in terms of the Friedmann equation and explain its possible origin. In Sec. 3, we review the spherical collapse model of structure formation in modified gravity, and compute the linear growth function and the linear density contrast at collapse time. Then, in Sec. 4, we study the effect of modified gravity on the SZ angular power spectrum. Observational constraints on the model parameters are discussed. Finally, we conclude in Sec. 5.

2 THE MODEL

The DGP braneworld was originally proposed as a model for recovering 4D gravity on the brane even in an infinitely large 5D Minkowski bulk (Dvali et al. 2000). In the DGP braneworld, gravity is modified at long distances, while standard 4D gravity is indeed reproduced at short distances (through a complicated nonlinear mechanism – e.g., Tanaka 2004; Koyama & Silva 2007; Lue & Starkman 2003; Lue et al. 2004), so that the model passes solar system and laboratory tests. Probably the most intriguing consequence of the DGP braneworld comes out in the modified Friedmann equation (Deffayet et al. 2002):

$$H^2 = \frac{8\pi G}{3}\rho \pm \frac{H}{r_c}. \quad (1)$$

Here, r_c is the crossover scale above which gravity looks 5D. With the upper (plus) sign and $r_c \sim H_0^{-1}$ (the present Hubble horizon scale), we have the “self-accelerating” solution which could be the origin of the current cosmic acceleration. In this paper, however, we do not assume that modified gravity is directly responsible for the accelerated expansion.

The model we consider is a phenomenological extension of the DGP braneworld described by the modified Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \pm \frac{H^{2\alpha}}{r_c^{2(1-\alpha)}}, \quad (2)$$

where $0 \leq \alpha < 1$. This is similar to the model of Dvali & Turner (2003), but we allow for a different sign of the last term and include the cosmological constant Λ explicitly (Afshordi et al. 2008). With the upper (respectively lower) choice of sign we use the terminology the self-accelerating (respectively normal) branch, though it is not *self*-accelerating in the upper sign case. Equation (2) with $\alpha = 1/2$ corresponds to the DGP cosmology (with a cosmological constant or the tension on the brane), while $\alpha = 1$ can be absorbed into a redefinition of the gravitational constant G . Expansion history of Λ CDM cosmology is recovered in the limit $\alpha \rightarrow 0$.

The modified Friedmann equation (2) can be recast in

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{m0}}{a^3} + \lambda \pm (r_c H_0)^{-2(1-\alpha)} \left(\frac{H}{H_0}\right)^{2\alpha}, \quad (3)$$

where

$$\lambda := 1 - \Omega_{m0} \mp (r_c H_0)^{-2(1-\alpha)}, \quad (4)$$

$\Omega_{m0} := 8\pi G\rho_0/(3H_0^2)$, and the present scale factor is chosen to be $a_0 = 1$. If $\Lambda(\propto \lambda) = 0$ and the accelerated expansion were supported by modified gravity in the self-accelerating branch, successful cosmology would require $r_c \sim H_0^{-1}$. In the present case, however, Λ is responsible for the accelerated expansion as in conventional cosmology, and therefore we are in principle allowed to take $r_c < H_0^{-1}$ (e.g., $r_c \sim 0.1H_0^{-1}$).¹ In particular, for sufficiently small α , expansion history is very close to that in standard Λ CDM cosmology even if we take relatively small r_c (Afshordi et al. 2008). Indeed, for $\alpha = 0.1$, $r_c H_0 = 0.4$, and $\Omega_{m0} = 0.26$, the dimensionless physical distance

$$E(a) := H_0 \int_a^1 \frac{da}{a^2 H(a)}, \quad (5)$$

differs from the corresponding Λ CDM result by less than one percent (Fig. 1).

Unfortunately, we do not have concrete higher dimensional models that account for Eq. (2). One possibility is that such modification could be derived from a higher codimension DGP model (Afshordi et al. 2008), as explained below. Let us consider a graviton propagator which is proportional to

$$\frac{1}{k^2 + k^{2\gamma}/r_c^{2(1-\gamma)}}. \quad (6)$$

This follows from a phenomenological model of modified gravity proposed in Dvali (2006) and is a power-law generalisation of the graviton propagator in the DGP braneworld. Since we are interested in long distance modification of gravity, we assume that $\gamma < 1$. The unitarity constraint requires $\gamma \geq 0$ (Dvali 2006). It can be seen that $\gamma = 1/2$ reproduces

¹ The normal branch does not admit acceleration without Λ from the beginning.

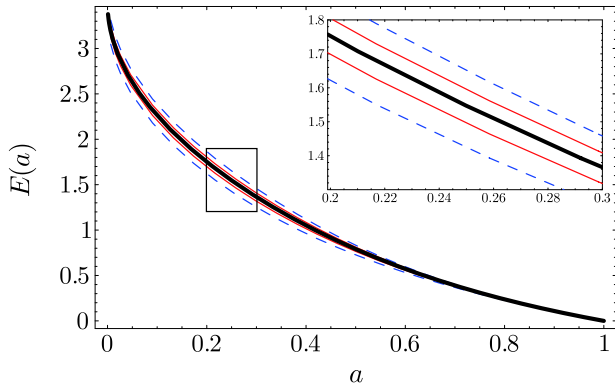


Figure 1. Dimensionless physical distance. Parameters are (from top to bottom): (1) $\alpha = 0.1$, $r_c H_0 = 0.4$; (2) $\alpha = 0.01$, $r_c H_0 = 0.2$; (3) Λ CDM; (4) $\alpha = 0.01$, $r_c H_0 = 0.2$; (5) $\alpha = 0.1$, $r_c H_0 = 0.4$. The cases (1) and (2) are for the normal branch and the cases (4) and (5) are for the self-accelerating branch. In all the cases $\Omega_{m0} = 0.26$.

the original DGP model and $\gamma = 1$ can be absorbed into a re-definition of G . From this observation, we may simply identify $\gamma = \alpha$. The propagator with $\gamma \ll 1$ has some connection to the so called cascading DGP braneworld (de Rham et al. 2008a,b),² and so the Friedmann equation with $\alpha \ll 1$ might be realised in such higher codimension models (see also Kaloper & Kiley 2007; Kobayashi 2008). However, we would like to stress that detailed analysis of higher codimension DGP models has yet to be undertaken and no braneworld models have been known so far that lead to Eq. (2). Therefore, we shall view Eq. (2) as a phenomenological starting point of our modified gravity.

3 STRUCTURE FORMATION IN MODIFIED GRAVITY

In the original DGP case, we have the covariant gravitational field equations that not only lead to the Friedmann equation (1) but also govern the behaviour of cosmological perturbations. As we do not know a complete set of field equations that underlies our phenomenologically extended model, we must *assume* something about the dynamics of cosmic inhomogeneities in the present case. Koyama (2006) clarified this issue by constructing simple covariant gravitational equations which give essentially the same Friedmann equation as Eq. (2). The gravitational equations contain an additional term called $E_{\mu\nu}$ in order to satisfy the Bianchi identity, which hinders to get closed form equations. In the original DGP braneworld, the evolution of the $E_{\mu\nu}$ term follows from the full 5D Einstein equations. In the absence of underlying theories for general α , one has to assume the structure of $E_{\mu\nu}$, which then determines the growth of structure. Koyama (2006) considered two possibilities: (i) weak gravity is described by the scalar-tensor theory, as in the original DGP model; (ii) the modified

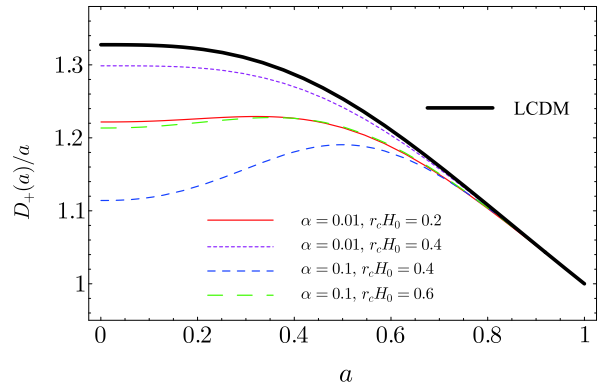


Figure 2. The growth function divided by the scale factor for different parameters. Plots are for the normal branch and $\Omega_{m0} = 0.26$.

gravity model respects Birkhoff's theorem (at least approximately). Afshordi et al. (2008) invoke the parameterized post-Friedmann framework (Hu & Sawicki 2007) and hence effectively take the first approach. In this paper, we employ the second approach and study nonlinear structure formation in modified gravity. The assumed Birkhoff's theorem allows us to use the modified Friedmann equation to track the nonlinear dynamics in a simple way, rather than introduce the extra scalar degree of freedom explicitly. The modified Friedmann equation reduces to the usual one in the high density regime, and in this sense we implement the nonlinear recovery of GR. In the cases investigated by Koyama (2006), the difference between the above two approach is small concerning the linear growth of perturbations.³ We come back to this issue in Appendix.

In this section, we compute the linear growth function and the linear density contrast for spherical collapse, which are the key quantities for the Press-Schechter formalism (Press & Schechter 1974) to predict the number density of clusters.

3.1 The growth equation

Following Schaefer & Koyama (2008), we study a spherical overdensity with matter density $\rho_c = \rho_c(t)$ and radius $R = R(t)$ in a background governed by the modified Friedmann equation (2), which can be recast in

$$H^2 = H_0^2 g(\xi), \quad \xi := \frac{8\pi G \rho}{3H_0^2}. \quad (7)$$

Differentiation with respect to t leads to

$$\frac{\ddot{a}}{a} = H_0^2 \left[g(\xi) - \frac{3}{2} \xi g'(\xi) \right], \quad (8)$$

where $\dot{a} := da/dt$ and $g' := dg/d\xi$.

² The codimension-two case in fact gives the propagator $\sim \ln k$ for small k .

³ Afshordi et al. (2008) focuses on the $\alpha \rightarrow 0$ limit, but in the PPF framework cosmological perturbations are still sensitive to r_c . On the other hand, the Birkhoff's theorem-based approach relies essentially on the Friedmann equation, and hence the effect of modified gravity vanishes in the $\alpha \rightarrow 0$ limit. Thus, the two approaches give different predictions at least in this limit.

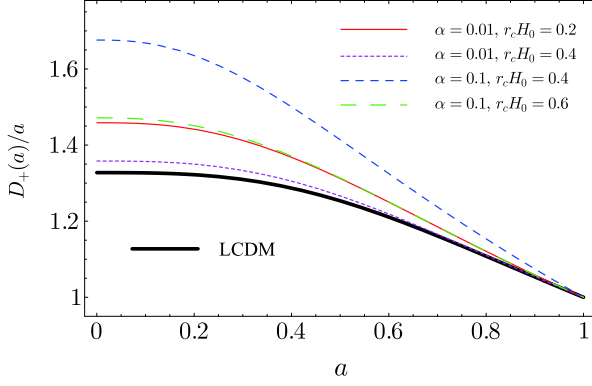


Figure 3. The growth function divided by the scale factor for different parameters. Plots are for the self-accelerating branch and $\Omega_{m0} = 0.26$.

Our central assumption is that *the modified gravity theory respects Birkhoff's theorem*. The dynamics of $R(t)$ is then described by

$$\frac{\ddot{R}}{R} = H_0^2 \left[g(\xi_c) - \frac{3}{2} \xi_c g'(\xi_c) \right], \quad (9)$$

with

$$\xi_c := \frac{8\pi G \rho_c}{3H_0^2}, \quad \rho_c \propto r^{-3}. \quad (10)$$

We define the overdensity as

$$\delta := \frac{\rho_c - \rho}{\rho}. \quad (11)$$

Upon linearisation the evolution equation for δ is given by

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G [g'(\xi) + 3\xi g''(\xi)] \rho \delta. \quad (12)$$

The growth function $D_+(a)$ is defined by $\delta(a, \mathbf{x}) = D_+(a)\delta(a_0, \mathbf{x})$. It follows from Eq. (12) that

$$\frac{d^2}{da^2} D_+ + \frac{f_1}{a} \frac{d}{da} D_+ = \frac{f_2}{a^2} D_+, \quad (13)$$

where

$$f_1 = 3 + \frac{d \ln H}{d \ln a}, \quad (14)$$

$$f_2 = 3 \frac{d \ln H}{d \ln a} + \left(\frac{d \ln H}{d \ln a} \right)^2 + \frac{a^2}{H} \frac{d^2 H}{da^2}. \quad (15)$$

The boundary condition is given by $D_+(0) = 0$ and $D_+(a_0) = 1$. Note that in standard cold dark matter dominated cosmology we find $D_+(a) = a$.

Figures 2 and 3 show typical behaviour of the growth function. For example, the difference between background expansion history $E(a)$ in modified gravity with $(\alpha, r_c H_0) = (0.01, 0.2)$ and that in the Λ CDM model is $\lesssim 3\%$, for which the growth functions differ by almost 10% (both in the self-accelerating and normal branches). In the case of the self-accelerating branch, the growth of perturbations is *suppressed* compared to the Λ CDM model, in agreement with the result of Schaefer & Koyama (2008).⁴ This is because the effective dark energy term becomes dominant earlier

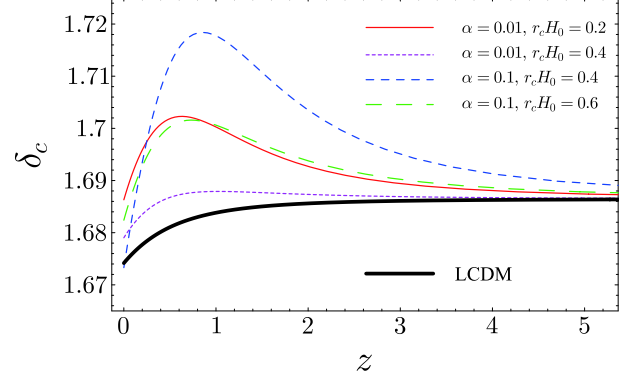


Figure 4. Overdensity at collapse time for different parameters. Plots are for the normal branch and $\Omega_{m0} = 0.26$.

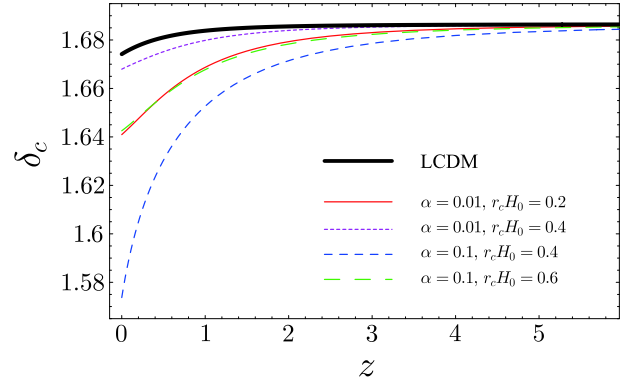


Figure 5. Overdensity at collapse time for different parameters. Plots are for the self-accelerating branch and $\Omega_{m0} = 0.26$.

in modified gravity than in the Λ CDM model due to the $+H^{2\alpha}$ term. In the normal branch, the effect of modified gravity works oppositely and the growth of perturbations is *enhanced* compared to the Λ CDM model. In the both cases, deviation from the Λ CDM result is larger for larger α and smaller r_c , as expected. Note here that the PPF approach predicts qualitatively the same result: structure is more evolved in the normal branch than in Λ CDM cosmology (Afshordi et al. 2008). Note also that qualitatively the same result, i.e., the suppressed (enhanced) growth function, is also found in the self-accelerating (normal) branch of the original DGP model ($\alpha = 1/2$) by solving the five-dimensional Einstein equations (Cardoso et al. 2008; Song 2008).

3.2 Spherical collapse

In order to study spherical collapse, it is convenient to use the quantities normalised by their values at turn-around time (Wang & Steinhardt 1998; Mota & van de Bruck 2004; Bartelmann et al. 2006). First, we define the normalised

⁴ Note that the normalization of the growth function here is such that $D_+(a_0) = 1$. Hence, smaller (respectively larger) $D_+(a)$ im-

plies a smaller (respectively larger) amplitude of the perturbation at $a < 1$ evolving into some fixed amplitude at $a = 1$, i.e., the growth is enhanced (suppressed).

scale factor and radius of the overdensity as follows:

$$x := a/a_{\text{ta}}, \quad y := R/R_{\text{ta}}. \quad (16)$$

We also define the dimensionless time $\tau := H_{\text{ta}} t$, where $H_{\text{ta}} := H(a_{\text{ta}})$ is the Hubble rate at turn around. Now the modified Friedmann equation can be written as

$$\frac{H^2}{H_{\text{ta}}^2} = \left(\frac{\dot{x}}{x}\right)^2 = \chi + 1 - \omega \pm (H_{\text{ta}} r_c)^{-2(1-\alpha)} \left[\left(\frac{\dot{x}}{x}\right)^{2\alpha} - 1 \right], \quad (17)$$

where $\omega := \Omega_{\text{m}0} H_0^2 / a_{\text{ta}}^3 H_{\text{ta}}^2$, $\chi := \omega/x^3$, and a dot here and hereafter denotes derivative with respect to τ . This equation can be rewritten as $(\dot{x}/x)^2 = h(\chi)$, or, equivalently,

$$\dot{x} = \sqrt{x^2 h\left(\frac{\omega}{x^3}\right)}. \quad (18)$$

Similarly to the previous calculation, one obtains the another Friedmann equation that describes the evolution of the overdensity patch:

$$\ddot{y} = y \left[h\left(\frac{\zeta\omega}{y^3}\right) - \frac{3}{2} \frac{\zeta\omega}{y^3} h'\left(\frac{\zeta\omega}{y^3}\right) \right], \quad (19)$$

where $h' := dh/d\chi$ and $\zeta := (\rho_c/\rho)|_{x=1}$. The boundary condition $y|_{x=0} = 0$, $\dot{y}|_{x=1} = 0$, and $y|_{x=1} = 1$ uniquely determines ζ . Equation (19) reduces to a first order differential equation by noticing that

$$\frac{d}{d\tau} \dot{y}^2 = \frac{d}{d\tau} \left[y^2 h\left(\frac{\zeta\omega}{y^3}\right) \right].$$

From this we obtain

$$\dot{y}^2 = y^2 h\left(\frac{\zeta\omega}{y^3}\right) - h(\zeta\omega), \quad (20)$$

where we fixed the integration constant by using the boundary condition $\dot{y} = 0$ at turn-around time ($y = 1$).

Since the background dynamics at early times is the same as that of the standard matter dominant universe, we may approximate $h(\omega/x^3) \simeq \omega/x^3$ for $x \ll 1$. Thus, at early times we simply have

$$\tau \simeq \frac{2}{3} \frac{x^{3/2}}{\sqrt{\omega}}. \quad (21)$$

Similarly, Eq. (20) reduces to

$$d\tau \simeq \sqrt{\frac{y}{\zeta\omega}} \left[1 + \frac{h(\zeta\omega)}{\zeta\omega} \frac{y}{2} \right] dy \quad \text{for } y \ll 1, \quad (22)$$

leading to

$$\tau \simeq \frac{2}{3} \frac{y^{3/2}}{\sqrt{\zeta\omega}} \left[1 + \frac{3}{10} \frac{h(\zeta\omega)}{\zeta\omega} y \right]. \quad (23)$$

The time evolution of the nonlinear overdensity, $\Delta := \zeta x^3 / y^3$, can be computed at early times by using

$$\Delta \simeq 1 + \frac{3}{5} \frac{h(\zeta\omega)}{\zeta\omega} y. \quad (24)$$

The linear density contrast at collapse time, δ_c , can be used to relate the nonlinear overdensity Δ with the density that would result from linear evolution and the same initial condition. Using the growth function, we have

$$\delta_c := \lim_{x \rightarrow 0} \frac{D_+(x_c)}{D_+(x)} [\Delta(x) - 1]. \quad (25)$$

We numerically computed $\delta_c(z)$ for various model parameters, and the results are illustrated in Figs. 4 and 5. We find that modified gravity does not change δ_c much: for example, δ_c in modified gravity with $(\alpha, r_c H_0) = (0.01, 0.2)$ differs from the Λ CDM prediction by 1 – 2% (both in the self-accelerating and normal branches).⁵

4 THE SUNYAEV-ZEL'DOVICH POWER SPECTRUM

In this section, we investigate the effect of modified gravity on the SZ angular power spectrum. Modification of gravity changes cluster number count through the modified growth function and critical density contrast. We concentrate on the case of the normal branch. The SZ angular power spectrum could be amplified in this branch because the growth function is enhanced relative to the Λ CDM model. The change in the critical density contrast is quite small, and hence it will give rise to only a negligible effect.

Modified gravity might in general affect the halo profiles assumed in the following discussion. This issue can be addressed by evaluating the Vainshtein radius, r_* , below which Einstein gravity is recovered. The Vainshtein radius in this particular model is given by $r_* = [r_c^{4(1-\alpha)} r_s]^{1/[1+4(1-\alpha)]} \simeq [r_c^4 r_s]^{1/5}$, where r_s is the Schwarzschild radius of the source (Dvali 2006). More explicitly, one has $r_* \sim 10^2 [(r_c H_0)^4 (M/M_\odot)]^{1/5}$ kpc, and for $r_c H_0 \sim 0.01$ and $M \sim 10^{14} M_\odot$, $r_* \sim 1$ Mpc. This estimate allows us to employ the halo profiles used in the context of standard gravity.

The computation of the SZ angular power spectrum is based on the halo formalism (Cole & Kaiser 1988; Makino & Suto 1993; Komatsu & Kitayama 1999; Komatsu & Seljak 2002). Since the one-halo Poisson term dominates the halo-halo correlation term on the scales we are interested in, we neglect the halo-halo correlation term. The SZ angular power spectrum is then given by

$$C_\ell^{SZ} = g_\nu^2 \int_0^{z_{\text{max}}} dz \frac{dV}{dz} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn(M, z)}{dM} |y_\ell(M, z)|^2, \quad (26)$$

where $V(z)$ is the comoving volume at z per steradian, $dn(M, z)/dM$ is the number density of clusters, $\tilde{y}_\ell(M, z)$ is the 2D Fourier transform of the projected Compton y -parameter, and g_ν is the spectral function of the SZ effect, which is given by

$$g_\nu = \frac{x^2 e^x}{(e^{2x} - 1)^2} \left[x \coth\left(\frac{1}{2}x\right) - 4 \right], \quad (27)$$

where $x = h\nu/k_B T_\gamma$ with the Boltzmann constant k_B and the CMB temperature T_γ .

As we are considering the spherical collapse scenario

⁵ It was reported that in the original DGP model with $\Lambda = 0$, the overdensity for spherical collapse decreases by almost 20% compared to conventional cosmology (Schaefer & Koyama 2008). However, the first version of the preprint (i.e., arXiv:0711.3129, v1) includes an error in the numerical calculations. We confirmed with the authors that δ_c differs from the standard one only by a small amount also in the case studied by Schaefer & Koyama (2008).

for the formation of halos, we utilise the Press-Schechter theory (Press & Schechter 1974),

$$\frac{dn(M, z)}{dM} = \sqrt{\frac{2}{\pi}} \frac{\rho}{M} \left[-\frac{\delta_c}{\sigma(M, z)} \frac{\partial \sigma}{\partial M} \right] \exp \left[-\frac{\delta_c^2}{2\sigma(M, z)} \right], \quad (28)$$

where δ_c is the critical over density which is obtained in the previous section, $\sigma(M, z)$ is the variance of the matter density field on the mass scale M . The variance σ is computed from the power spectrum of linear matter density fluctuations with the top hat filter,

$$\sigma^2(M, z) = \int dk k^2 P_m(k, z) W(kR), \quad (29)$$

where $W(kR)$ is the top hat window function and R is the scale which corresponds to $M (= 4\pi\rho R^3/3)$. We calculate the linear power spectrum $P_m(k, z)$ in the modified gravity model by using the growth function obtained in the previous section and the initial condition for the curvature perturbation and the scalar spectral index, $\Delta_{\mathcal{R}}^2 = 2.41 \times 10^{-9}$, $n = 0.96$, given by WMAP (Komatsu et al. 2009).

The 2D Fourier transform of the projected Compton y -parameter is given by

$$y_\ell = \frac{4\pi r_s}{\ell_s^2} \int_0^\infty dx x^2 y_{3d}(x) \frac{\sin(\ell x/\ell_s)}{\ell x/\ell_s}, \quad (30)$$

where y_{3d} is the radial profile of the Compton y -parameter,

$$y_{3d}(x) = \frac{\sigma_T}{m_e} n_e(x) k_B T_e(x), \quad (31)$$

with σ_T being the Thomson cross section and m_e the electron mass. ℓ_s is the angular wavenumber corresponding to r_s , $\ell_s = D_A/r_s$, with D_A being the angular diameter distance.

For the electron density profile n_e and the temperature profile T_e , we use the result of Komatsu & Seljak (2002), which is based on the NFW dark matter density profile (Navarro et al. 1997). The NFW dark matter density profile is given by

$$\rho_{\text{DM}}(x) = \frac{\rho_s}{x(1+x)^2}, \quad (32)$$

where $x := r/r_s$ where r_s is a scale radius, and ρ_s is a scale density. The scale radius r_s is related to the virial radius by the concentration parameter c as

$$r_s(M, z) = \frac{r_{\text{vir}}(M, z)}{c(M, z)}. \quad (33)$$

We use the concentration parameter of Komatsu & Seljak (2002),

$$c(M, z) \approx \frac{10}{1+z} \left[\frac{M}{M_*(0)} \right]^{-0.2}, \quad (34)$$

where $M_*(0)$ is a solution to $\sigma(M, 0) = \delta_c$ at the redshift $z = 0$.

We adopt the spherical collapsed model to obtain the virial radius,

$$r_{\text{vir}}(M, z) = \frac{3M}{4\pi\Delta_v(z)\rho(z)}, \quad (35)$$

where $\Delta_v(z)$ is the virialised overdensity at z . Modified gravity changes the virialised overdensity, which we compute following Schmidt et al. (2009). For example, the modified virial overdensity at $z = 0$ is $\Delta_v = 270$ for $(\alpha, r_c H_0) = (0.1, 0.3)$ and $\Delta_v = 300$ for $(\alpha, r_c H_0) = (0.1, 0.4)$, while

$\Delta_v = 370$ in the Λ CDM model. Thus, we find that the actual density in a virialised halo, $\Delta_v(z)\rho(z)$, is lower in modified gravity than in the Λ CDM model.

In order to obtain the profiles of the electron density and temperature, Komatsu & Seljak (2002) assumed three things: (i) the electron gas is in hydrostatic equilibrium in the dark matter potential; (ii) the electron gas density follows the dark matter density in the outer part of the halo; (iii) the equation of state of the electron gas is polytropic, $P_e \propto \rho_e^\gamma$, where P_e , ρ_e and γ are the electron gas pressure, the gas density and the polytropic index, respectively. Under these assumptions, the electron number density and temperature profiles are simply given by

$$n_e = n_{ec} F(x), \quad (36)$$

$$T_e = T_{ec} F^{\gamma-1}(x). \quad (37)$$

Here, n_{ec} and T_{ec} are the electron density and temperature at the centre, respectively, and the dimensionless profile $F(x)$ is written as

$$F(x) = \left\{ 1 - A \left[1 - \frac{\ln(1+c)}{x} \right] \right\}^{1/(\gamma-1)}, \quad (38)$$

where the coefficient A is defined as

$$A := 3\eta_c^{-1} \frac{\gamma-1}{\gamma} \left[\frac{\ln(1+c)}{c} - \frac{1}{1+c} \right]^{-1}. \quad (39)$$

The central electron density n_{ec} and the temperature T_{ec} are given by

$$n_{ec} = 3.01 \left(\frac{M}{10^{14} M_\odot} \right) \left(\frac{r_{\text{vir}}}{1 \text{ Mpc}} \right)^{-3} \times \frac{c}{(1+c)^2} \left[\ln(1+c) - \frac{c}{1+c} \right]^{-1} F^{-1}(c) \text{ cm}^{-3}, \quad (40)$$

$$T_{ec} = 0.88 \eta_c \left[\frac{M/(10^{14} h^{-1} M_\odot)}{r_{\text{vir}}/(1 h^{-1} \text{ Mpc})} \right] \text{ keV}. \quad (41)$$

For γ and η_c , Komatsu & Seljak (2002) provided the following useful fitting formulas:

$$\gamma = 1.137 + 8.94 \times 10^{-2} \ln(c/5) - 3.68 \times 10^{-3} (c-5), \quad (42)$$

$$\eta_c = 2.235 + 0.202(c-5) - 1.16 \times 10^{-3} (c-5)^2. \quad (43)$$

Figure 6 shows the SZ power spectrum in the modified gravity model. We take three sets of the model parameters: $(\alpha, r_c H_0) = (0.1, 0.2)$, $(0.1, 0.3)$, and $(0.1, 0.4)$, giving $\sigma_8 = 1.2$, 1.0 and 0.9 , respectively (The initial density power spectra are the same for all of these parameter sets). For comparison, we plot in Fig. 6 the SZ power spectra in the Λ CDM models with $\sigma_8 = 0.77$ (our fiducial model) and with $\sigma_8 = 1.0$. One sees that the amplitude of the peak in modified gravity is lower than that in the Λ CDM model with the same σ_8 . This is because the actual density in a virialised halo is lower in modified gravity than in the Λ CDM model.

Moreover, modified gravity shows the damping of the SZ power spectrum on small scales and the amplification on large scales, because the contribution from halos at high redshifts in modified gravity is smaller than that in the Λ CDM model. The redshift contribution for different ℓ is shown in Fig. 7. The growth of matter density fluctuations in the modified gravity model is rapid at late times compared with the growth in the Λ CDM model with the same σ_8 . As a result, the formation of halos delays, leading to the large contri-

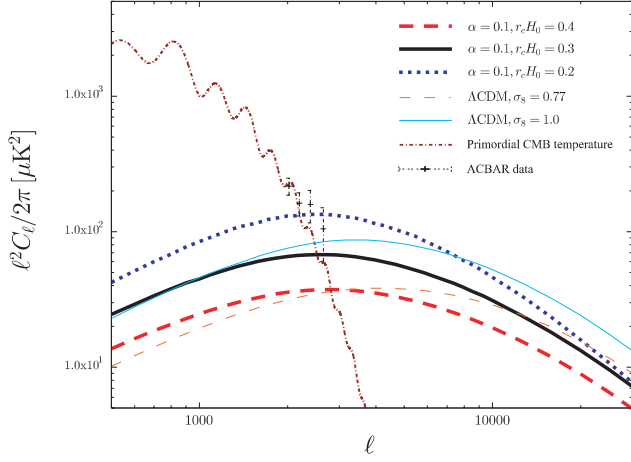


Figure 6. SZ angular power spectra for different modified gravity parameters. The dotted, solid, and dashed lines represent SZ power spectra for $(\alpha, r_c H_0) = (0.1, 0.2)$, $(0.1, 0.3)$, and $(0.1, 0.4)$, respectively. The SZ angular power spectra for $\sigma_8 = 1.0$ and $\sigma_8 = 0.77$ in the Λ CDM model are shown as the thin solid and thin dashed lines, respectively. For references, we plot the primordial CMB temperature angular power spectrum in our fiducial Λ CDM model and the ACBAR data.

bution from low redshifts and small contribution from high redshifts.

The SZ power spectrum is constrained by the CMB observations on small scales. In the Λ CDM model the ACBAR data at $\ell \sim 2500$ gives the constraint on the SZ power spectrum in terms of σ_8 : $\sigma_8 < 1$ (Kuo et al. 2007). The peak amplitude of the SZ power spectrum is lower in modified gravity than in the Λ CDM model with the same σ_8 . This result then yields the constraint from the ACBAR data $\sigma_8 < 1.2$ in modified gravity. Figure 8 shows σ_8 as a function of the model parameters α and $r_c H_0$. In the limit $r_c \rightarrow \infty$, σ_8 reduces to the value in our fiducial model, $\sigma_8 = 0.77$. From the ACBAR experiment, the parameter region below the $\sigma_8 = 1.2$ line in Fig. 8 is excluded.

Stronger constraints can be obtained from the small angular scale CMB measurements by the QUaD experiment (Friedman et al. 2009). The QUaD telescope measured the CMB temperature anisotropy in the multipole range $2000 < \ell < 3000$ at 150 GHz. The QUaD team reported no strong evidence of the SZ effect and the result is consistent with the WMAP data which yields $\sigma_8 \sim 0.8$. Therefore, using the result of the QUaD experiment to constrain σ_8 , the modified gravity parameters are more strongly restricted than in the case of the ACBAR data. The constraint from the QUaD data gives $\sigma_8 \sim 0.9$ in modified gravity.

5 CONCLUSIONS

In this paper, we have explored observational consequences of structure formation in modified gravity. The model considered is a phenomenological extension of the DGP braneworld, and modification to the standard Λ CDM model is characterized by the additional term $\pm H^{2\alpha}/r_c^{2(1-\alpha)}$ in the Friedmann equation. In the case of $\alpha = 1/2$, the term arises from the five-dimensional effect, but in this work α and r_c were assumed to be free parameters.

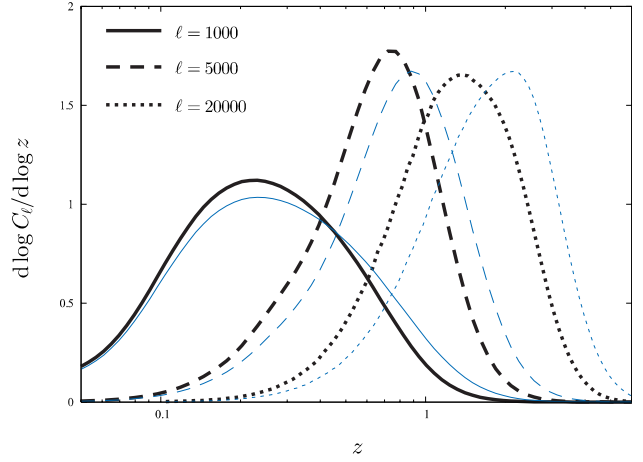


Figure 7. Distribution of the redshift contribution of the SZ angular power spectrum for different ℓ modes. We set the modified gravity parameters $\alpha = 0.1$ and $r_c H_0 = 0.3$. The solid, the dashed, and the dotted lines represent the distributions for $\ell = 1000$, $\ell = 5000$, and $\ell = 20000$, respectively. For comparison, we plot the distributions for the Λ CDM model with $\sigma_8 = 1.0$ as thin lines.

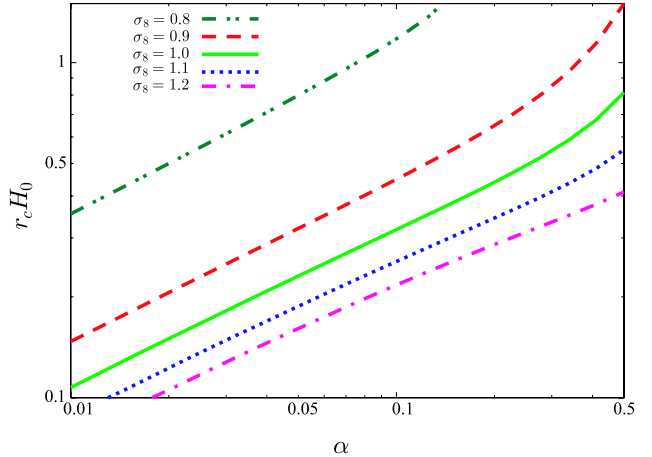


Figure 8. The amplitude σ_8 as a function of modified gravity parameter α and r_c . The dashed-dotted-dotted, dashed, solid, dotted, dashed-dotted lines represent $\sigma_8 = 0.8, 0.9, 1.0, 1.1$ and 1.2 .

First, we have studied the spherical collapse model of nonlinear structure formation in modified gravity. It was found that change in the growth function is relatively large, but the linear density contrast for spherical collapse undergoes very small modification. For the self-accelerating branch, the growth of perturbations is suppressed compared to the Λ CDM model, which confirms qualitatively the result of Schaefer & Koyama (2008). For the normal branch, the growth of perturbations is enhanced compared to the Λ CDM model.

Focusing on the normal branch, we then investigated the effect of modified gravity on the SZ angular power spectrum. The enhanced growth function in the normal branch results in the amplification of σ_8 . However, the modification to the SZ power spectrum is rather nontrivial. The peak

amplitude of the SZ angular power spectrum is lower in modified gravity than in the Λ CDM model with the same σ_8 , because halos are virialised at lower density in modified gravity than in the Λ CDM model. In addition to this, modified gravity shows the damping of the SZ power spectrum on small scales.

We confronted the modified SZ spectrum with CMB observations on small scales. Observational constraints can be read off from Fig. 8. The ACBAR experiment gives the constraint on the SZ power spectrum which translates to $\sigma_8 = 1.2$ in modified gravity. From this we put constraints to the parameters of the modified gravity model: for example, we have $r_c H_0 > 0.4$ for $\alpha = 0.5$ and $r_c H_0 > 0.2$ for $\alpha = 0.1$. The QUaD data gives severer constraints on the parameters: $\sigma_8 = 0.9$, leading to $r_c H_0 > 0.4$ for $\alpha = 0.1$.

ACKNOWLEDGMENTS

We would like to thank Björn Malte Schäfer and Kazuya Koyama for helpful correspondences and Naoshi Sugiyama for useful comments. We also would like to thank an anonymous referee for useful comments to improve the paper. TK is supported by the JSPS under Contact No. 19-4199.

APPENDIX A: COMPARISON WITH DIFFERENT APPROACHES

In the phenomenologically extended version of the DGP model, different assumptions can be made in computing the growth of perturbations. Instead of using Birkhoff's theorem, we may assume that the behaviour of perturbations is governed by a scalar-tensor theory. Following Koyama (2006), we now compare the linear evolution of perturbations obtained by these two approaches. If one does not assume Birkhoff's theorem but employs a scalar-tensor theory as an effective theory for perturbations, the evolution of δ will be described by (Koyama 2006)

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \left(1 + \frac{1}{3\beta}\right) \rho\delta, \quad (\text{A1})$$

where

$$\beta := 1 + \frac{(r_c H)^{2(1-\alpha)}}{\alpha} \left[1 + \frac{2}{3} \frac{(1-\alpha)\dot{H}}{H^2}\right]. \quad (\text{A2})$$

Note that this is the expression for the normal branch. We denote by $D_+^*(a)$ the growth function calculated from this equation. The difference between D_+^* and D_+ obtained in the main text, $(D_+^* - D_+)/ (D_+^* + D_+)$, is presented in Fig. A1. As can be seen, there is only a few percent difference between the two approaches concerning the linear growth of perturbations. Developing nonlinear theory with the violation of Birkhoff's theorem taken into account is beyond the scope of the present paper.

REFERENCES

Afshordi N., Geshnizjani G., Khoury J., 2008, arXiv, 0812.2244
 Bartelmann M., Doran M., Wetterich C., 2006, *Astron. Astrophys.*, 454, 27

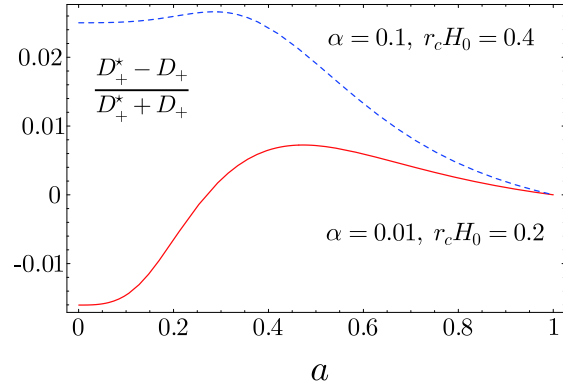


Figure A1. Comparison with the growth functions obtained by the different approaches.

Bekenstein J. D., 2004, *Phys. Rev.*, D70, 083509
 Cardoso A., Koyama K., Seahra S. S., Silva F. P., 2008, *Phys. Rev.*, D77, 083512
 Cole S., Kaiser N., 1988, *Mon. Not. Roy. Astron. Soc.*, 233, 637
 de Rham C., et al., 2008a, *JCAP*, 0802, 011
 de Rham C., et al., 2008b, *Phys. Rev. Lett.*, 100, 251603
 Deffayet C., Dvali G. R., Gabadadze G., 2002, *Phys. Rev.*, D65, 044023
 Dvali G., 2006, *New J. Phys.*, 8, 326
 Dvali G., Turner M. S., 2003, arXiv, astro-ph/0301510
 Dvali G. R., Gabadadze G., Porrati M., 2000, *Phys. Lett.*, B485, 208
 Friedman R. B., et al., 2009, arXiv, 0901.4334
 Hu W., Sawicki I., 2007, *Phys. Rev.*, D76, 104043
 Kaloper N., Kiley D., 2007, *JHEP*, 05, 045
 Khoury J., Wyman M., 2009, arXiv, 0903.1292
 Kobayashi T., 2008, *Phys. Rev.*, D78, 084018
 Komatsu E., et al., 2009, *Astrophys. J. S.*, 180, 330
 Komatsu E., Kitayama T., 1999, *Astrophys. J.*, 526, L1
 Komatsu E., Seljak U., 2002, *Mon. Not. Roy. Astron. Soc.*, 336, 1256
 Koyama K., 2006, *JCAP*, 0603, 017
 Koyama K., Silva F. P., 2007, *Phys. Rev.*, D75, 084040
 Kuo C. L., et al., 2007, *Astrophys. J.*, 664, 687
 Lue A., Scoccimarro R., Starkman G. D., 2004, *Phys. Rev.*, D69, 124015
 Lue A., Starkman G., 2003, *Phys. Rev.*, D67, 064002
 Makino N., Suto Y., 1993, *Astrophys. J.*, 405, 1
 Martino M. C., Stabenau H. F., Sheth R. K., 2008, arXiv, 0812.0200
 Milgrom M., 1983, *Astrophys. J.*, 270, 384
 Mota D. F., van de Bruck C., 2004, *Astron. Astrophys.*, 421, 71
 Navarro J. F., Frenk C. S., White S. D. M., 1997, *ApJ*, 490, 493
 Press W. H., Schechter P., 1974, *Astrophys. J.*, 187, 425
 Schäfer B. M., Koyama K., 2008, *Mon. Not. Roy. Astron. Soc.*, 385, 411
 Schmidt F., 2008, *Phys. Rev.*, D78, 043002
 Schmidt, F. and Lima, M. and Oyaizu, H. and Hu, W., 2009, *Phys. Rev.*, D79, 083518
 Sealfon C., Verde L., Jimenez R., 2005, *Phys. Rev.*, D71, 083004

- Shirata A., Shiromizu T., Yoshida N., Suto Y., 2005, Phys. Rev., D71, 064030
- Shirata A., Suto Y., Hikage C., Shiromizu T., Yoshida N., 2007, Phys. Rev., D76, 044026
- Song Y.-S., 2008, Phys. Rev., D77, 124031
- Sotiriou T. P., Faraoni V., 2008, arXiv, 0805.1726
- Stabenau H. F., Jain B., 2006, Phys. Rev., D74, 084007
- Tanaka T., 2004, Phys. Rev., D69, 024001
- Thomas S. A., Abdalla F. B., Weller J., 2008, arXiv, 0810.4863
- Wang L.-M., Steinhardt P. J., 1998, Astrophys. J., 508, 483